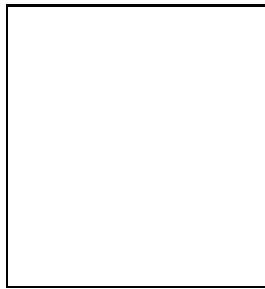


**The status of NLL BFKL <sup>a</sup>**G.P. Salam  
*TH Division, CERN, CH-1211 Genève 23.*

This talk summarises the current status of the NLL corrections to BFKL physics and discusses the question of small- $x$  factorisation.

**1 Introduction and total cross sections**

Understanding perturbative high-energy hadronic scattering is one of the fundamental problems of QCD, and one which has seen considerable progress in the past couple of years.

The reaction under consideration is that of the scattering of objects with some transverse scale  $Q^2$  at a centre-of-mass energy  $\sqrt{s}$ , in the limit  $s \gg Q^2 \gg \Lambda_{\text{QCD}}^2$ . This was first examined in the mid 1970's with the resummation of the leading logarithmic (LL) terms  $(\alpha_s \ln s)^n$  to give the result that the cross section  $\sigma \sim s^{4 \ln 2 \alpha_s N_C / \pi}$ . This is known as the BFKL pomeron.<sup>1</sup> This result however seems to be in contradiction with numerous experimental results, which invariably indicate that while there is a rise of the cross section at large  $s$ , the power is somewhat lower, of the order of 0.3 as opposed to the 0.5 expected from the LL calculation (for  $\alpha_s \simeq 0.2$ ).

It was expected that this discrepancy would be resolved by the inclusion of the next-to-leading (NLL) corrections  $\alpha_s(\alpha_s \ln s)^n$ , but it turns out that they modify the power  $\omega$  as follows:<sup>2</sup>

$$\omega = 2.65\alpha_s - 16.3\alpha_s^2 \simeq -0.12 \quad (\text{for } \alpha_s = 0.2), \quad (1)$$

which is just as incompatible with the data as the LL result. Closer inspection reveals that the NLL corrections, taken literally, also imply negative cross sections for the scattering of two objects of different transverse sizes<sup>3</sup> (essentially, in such a collinear limit the convergence becomes even worse).

There have been several significantly different attempts to explain the large size of the corrections and to thus estimate yet higher-order corrections with the aim of obtaining stable

---

<sup>a</sup>Talk presented at the XXXVth Rencontres de Moriond, QCD and High Energy Hadronic Interactions, Les Arcs, France, March 18-25, 2000.

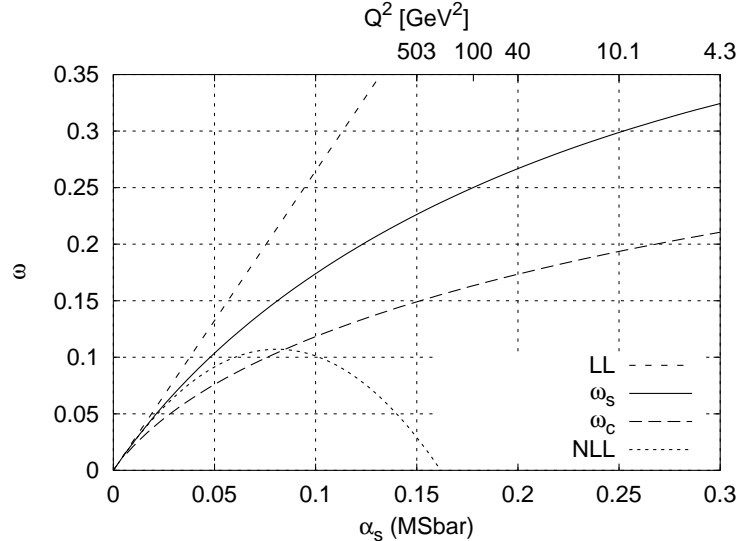


Figure 1: The prediction for the BFKL power as a function of  $\alpha_s$  at LL, NLL and after the all-order inclusion of collinearly enhanced terms ( $\omega_s$ ). The uncertainty on  $\omega_s$  is of the order of 10%. The curve labelled  $\omega_c$  is the BFKL power for splitting functions. The results, shown for  $n_f = 0$ , depend only weakly on  $n_f$ .

predictions. One involves the idea of a minimum rapidity gap between emissions,<sup>4</sup> while another argues that a meaningful answer can be obtained using a more ‘natural’ renormalisation scheme coupled with BLM resummation.<sup>5</sup> Both suffer from instabilities (with respect to the choice of the rapidity gap and the renormalisation scheme, respectively) and have difficulty solving the problem of the negative cross sections, essentially because they do not take into account the worsening of the convergence in the collinear limit.

An alternative approach indeed starts from a consideration of the collinear limit, and observes that a large part of the NLL corrections (even for non-collinear scattering) come precisely from terms that are enhanced in the collinear limit. These can be resummed by requiring that the cross section satisfy the renormalisation group properties in both collinear limits (there are two collinear limits corresponding to which of the scattering objects is the smaller).<sup>6</sup> One finds that the problem of negative cross sections goes away completely and that the power of the cross section growth is relatively stable with respect changes of scheme. The result for the power as a function of  $\alpha_s$  is shown in fig. 1.

The cleanest (most inclusive) measurement of the BFKL power comes from  $\gamma^*\gamma^*$  scattering, by the L3 collaboration<sup>7</sup> at scales  $Q^2 = 3.5$  and  $14.5$  GeV<sup>2</sup>. They quote a value for the power of  $\omega = 0.37 \pm 0.04$ . The quoted error is probably an underestimate: in the fit procedure the normalisation is fixed to be the LL value, whereas one should fit also for the normalisation because it too is liable to be subject to significant (but as yet uncalculated) higher-order corrections. A more realistic error is about three times larger. There are also systematic errors associated with the functional form used to fit for the power. So the measurement is roughly in accord with the theoretical expectation which goes from 0.29 to 0.33 for this range of scales (for comparison the LL power is in the range 0.6 to 0.8), but a satisfactory comparison requires, on the experimental front, higher precision data and larger energies (*e.g.* at the NLC), and on the theoretical front a calculation of the expected normalisation (*i.e.* the impact factors to NLL order).

## 2 Scaling violations

So far we have discussed the total cross section in a context where both scattering objects are perturbative. An important situation is that in which only one of the objects is perturbative,

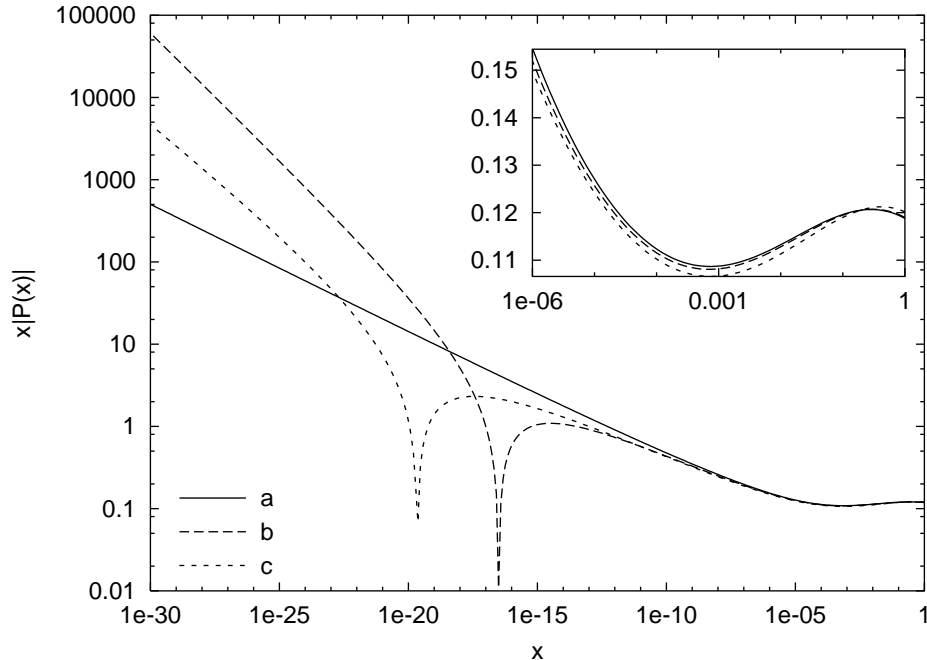


Figure 2: The splitting function in LL BFKL with running coupling for three different sets (a,b,c) of infra-red regularisation and initial transverse scale. (Shown for  $\alpha_s = 0.126$ .) The inset shows the same curves in a more limited range of  $x$  values.

namely deep inelastic scattering. Since the other object is non-perturbative the calculation of the total cross section is beyond perturbation theory. When the  $s$  is of the same order as  $Q^2$  (*i.e.*  $x$  not too small) we know however that the cross section can be factorised into non-perturbative parton distributions convoluted with perturbative coefficient functions, and the scaling violations of the parton distributions can be calculated perturbatively through the DGLAP equations.<sup>8</sup> The intuitive justification for this picture is that emissions are ordered in transverse scale, so that the non-perturbative parton distribution contains emissions only below a given scale  $Q_0^2$ , while the coefficient function (and parton distribution evolution) involves only subsequent emissions, which are all above  $Q_0^2$ .

However in high-energy scattering (*i.e.* at small  $x$ ) it has long been known that emissions are not ordered in transverse momentum. This led to the suggestion<sup>9</sup> that the factorisation which holds at moderate  $x$  might break down at small  $x$ . (Recently, on this basis, it has been argued that small- $x$  splitting functions are beyond calculation and can at best be fitted.<sup>10</sup>)

Last summer an explicit counterexample was presented,<sup>11</sup> namely a toy model which retained the fundamental property of BFKL evolution, namely emissions which are not ordered in transverse scale, but in which the property of factorisation could be demonstrated for all  $x$ . Recently, numerical methods have been developed<sup>12</sup> which allow the study of factorisation in the full BFKL equation (for the time being at LL, with running coupling). In this approach one solves the BFKL equation to obtain a gluon distribution as a function of  $x$  and  $Q^2$ . This distribution depends intrinsically on the regularisation of  $\alpha_s$  in the infra-red (IR) and the initial transverse scale. One then performs a deconvolution of the scaling violations from the gluon distribution, to obtain an effective splitting function.

The results of this procedure are shown in fig. 2, which shows the effective splitting function for three different sets of infra-red regularisations. One sees that the splitting function is independent of the IR regularisation, except at very small  $x$ , where a non-perturbative component takes over. More detailed investigation reveals that this second component is higher-twist, and

despite its much stronger  $x$  dependence, the scaling violations themselves can be predicted to within a relative higher-twist term which is not  $x$  enhanced, thus confirming that factorisation holds even at small  $x$ .

A rough explanation of why factorisation holds is the following: the fact that the evolution is not ordered in transverse momentum means that the separation into a parton distribution and coefficient or splitting functions is less trivial. If one chooses to separate the two at some scale  $Q_0$ , then the parton distribution now depends on emissions both below and above  $Q_0^2$ . But significantly, the splitting and coefficient functions turn out to still only depend on emissions above  $Q_0^2$ .

This brings us to the question of what exactly the small- $x$  splitting functions look like. A fundamental property is that the power-growth only sets in at relatively small  $x$ . It is also significantly weaker than what would be observed in total cross sections at the same scale. This is the reason for the presence of two powers in fig. 1:  $\omega_s$  is that which applies to total cross sections, while  $\omega_c$  is that which is relevant for splitting functions. These two points (which have been noted also by Thorne<sup>13</sup>) mean that for the  $x$  values that are relevant at HERA, the small- $x$  resummed splitting functions are probably quite similar to the DGLAP splitting functions, explaining the unexpected success of the latter in reproducing the observed scaling violations.

## Acknowledgements

I wish to thank Marcello Ciafaloni and Dimitri Colferai (in collaboration with whom many of the results presented here have been obtained) for discussions. This work was supported in part by the E.U. QCDNET contract FMRX-CT98-0194.

## References

1. L.N. Lipatov, *Sov. J. Nucl. Phys.* **23** (1976) 338; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov. Phys. JETP* **44** (1976) 443; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov. Phys. JETP* **45** (1977) 199; Ya. Balitskii and L.N. Lipatov, *Sov. J. Nucl. Phys.* **28** (1978) 822.
2. V.S. Fadin and L.N. Lipatov, *Phys. Lett.* **B 429** (1998) 127 [hep-ph/9802290]; M. Ciafaloni and G. Camici, *Phys. Lett.* **B 430** (1998) 349 [hep-ph/9803389]; references therein.
3. D.A. Ross, *Phys. Lett.* **B 431** (1998) 161 [hep-ph/9804332].
4. C. Schmidt, *Phys. Rev.* **D 60** (1999) 074003 [hep-ph/9901397].
5. S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, *JETP Lett.* **70** (1999) 155 [hep-ph/9901229].
6. G.P. Salam, *JHEP* **07** (1998) 19 [hep-ph/9806482]; M. Ciafaloni and D. Colferai, *Phys. Lett.* **B 452** (1999) 372 [hep-ph/9812366]; M. Ciafaloni, D. Colferai and G.P. Salam, *Phys. Rev.* **D 60** (1999) 114036 [hep-ph/9905566].
7. L3 Collaboration (M. Acciarri et al.), *Phys. Lett.* **B 453** (1999) 333; C.H. LIN, these proceedings.
8. V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 438; G. Altarelli and G. Parisi, *Nucl. Phys.* **B 126** (1977) 298; Yu.L. Dokshitzer, *Sov. Phys. JETP* **46** (1977) 641.
9. A.H. Mueller, *Phys. Lett.* **B 396** (1997) 251 [hep-ph/9612251].
10. G. Altarelli, R.D. Ball and S. Forte, hep-ph/9911273.
11. M. Ciafaloni, D. Colferai and G.P. Salam, *JHEP* **10** (1999) 017 [hep-ph/9907409].
12. M. Ciafaloni, D. Colferai, G.P. Salam and M. Taiuti, in preparation.
13. R.S. Thorne, *Phys. Lett.* **B 474** (2000) 372 [hep-ph/9912284].